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Chiral gravity as a covariant formulation of massive gravity

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Abstract

We present a covariant nonlinear completion of the Fierz–Pauli (FP) mass term for the graviton. The starting observation is that the FP mass is immediately obtained by expanding the cosmological constant term, i.e. the determinant of the vielbein, around Minkowski space to second order in the vielbein perturbations. Since this is an unstable expansion in the standard case, we consider an extended theory of gravity which describes two vielbeins that give rise to chiral spin–connections (consequently, fermions of a definite chirality only couple to one of the gravitational sectors). As for Einstein gravity with a cosmological constant, a single fine–tuning is needed to recover a Minkowski background; the two sectors then differ only by a constant conformal factor. The spectrum of this theory consists of a massless and a massive graviton, with FP mass term. The theory possesses interesting limits in which only the massive graviton is coupled to matter at the linearized level.

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1 Introduction

There is compelling evidence that the expansion of the Universe is presently accelerating [1]. The simplest cause for this effect, a vacuum energy $\Lambda \sim 10^{-120}\, M_P^4$, is so unnaturally small that it is worth searching for alternative explanations. The most straightforward possibility is that, due to some (unknown) symmetry, $\Lambda = 0$, and that the present acceleration is instead due to some (quintessence) field whose equation of state is sufficiently close to the one of vacuum, $w \leq -0.7$ [1]. An alternative, more drastic, possibility, is that the present accelerated expansion signals a modification of standard gravity at very large scales. A well–studied modification is massive gravity, which is expected to be weaker at distances larger than the inverse graviton mass m_g^{-1} by Yukawa suppression. If the graviton mass is comparable with the present Hubble parameter H_0, this may lead to accelerated expansion at the largest observable scales. A different road to modify the gravitational interactions is to introduce a "nonstandard" form of matter, which, once coupled to a conventional gravitational sector, can change the properties of the graviton. One recent example is the theory of ghost condensation [2], where the Lorentz symmetry is broken by the gradient of a scalar field. [3] Although these proposals are quite speculative, they are certainly suggestive directions worth further investigations. The quest for consistent modifications of standard gravity is by itself a very interesting and nontrivial theoretical subject, which has indeed drawn considerable attention both in the past and at present.

The structure of the mass term for the graviton was investigated by Fierz and Pauli [4] already in 1939. They showed that the only Lorentz–invariant ghost–free mass term for a graviton in Minkowski background [4] is

$$\frac{1}{2}\, m_g^2\, \eta^{\mu\nu}\, \eta^{\alpha\beta} \Big[h_{\mu\nu}\, h_{\alpha\beta} - h_{\mu\alpha}\, h_{\nu\beta} \Big] \;, \tag{1}$$

where $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ defines the metric perturbations. Because this Fierz–Pauli (FP) term (1) is not covariant, the graviton acquires three additional degrees of freedom, that are instead gauge modes for standard gravity. The divergence–free vector components do not couple to conserved sources in the massless limit. However, the third (longitudinal) component is sourced by the trace of the energy–momentum tensor, and it does not decouple in this limit. Consequently, the graviton propagator exhibits a discontinuity at the linearized level between the massless and the massive case [6]. The main phenomenological consequence of this discontinuity would be a too large modification of the bending of the light from the sun. However, nonlinear effects drastically change this picture: As realized in [7] (and re–examined in [8, 9]), gravity mediated by a massive graviton becomes strong at macroscopic distances. By extending the Schwarzschild solution to the FP case, one can show that, while the linearized (one graviton exchange) approximation is indeed discontinuous, the full nonperturbative solution has a smooth $m_g \to 0$ limit [7], at least at some finite range of distances from the source. It is however unclear whether the solution found in [7] can be extended into a regular solution from the source up to infinity; some specific nonlinear completions of (1) have been analyzed – partially through numerical calculations – in [10], and the solutions were found to develop singularities at finite distance from the source. However, it is hard to see whether this conclusion is true for arbitrary completions.

Therefore, the FP mass term (1) can not be ruled out by the discontinuity present at the linear analysis. Unfortunately, going beyond the linear regime one faces the problem of the strong sensitivity to the completion of massive gravity. This is particularly true if we think to massive gravity as an

[3] The phenomenology of this model in the physically relevant cases has been studied in [3].
[4] As discussed in [5], a richer structure of ghost–free mass terms is possible if one is willing to give up Lorentz invariance.

effective field theory, where all higher order interactions not forbidden by any symmetry should be included. Among the symmetries restricting these higher order terms, one can easily argue that the nonlinear completion should be covariant. As shown in [11] (see also [12]) FP gravity has a clear instability at the nonlinear level, related to the fact that, with the FP term present, the Hamiltonian of the system is not positive semi–definite. This problem seems to persist for generic nonlinear completions of (1). In a covariant theory, like the Einstein gravity, the Hamiltonian defines a constraint, so that this instability is not present from the outset. Hence, one may expect that a covariant non–linear completion of the FP mass term does not suffer from this specific nonlinear instability.

When one is considering modifications of gravity, one may contemplate other possibilities beside a possible graviton mass. One rather exotic question is whether a theory of gravity can make a distinction between fermions of different chirality. This is not an unnatural question in light of the Standard Model (SM) of particle physics, since the Electroweak interaction couples differently to left– and right–handed quarks and leptons. This is technically implemented using a chiral gauge connection and exploiting that some fermions are $\mathrm{SU}(2)_L$ doublets while others are singlet. Applying such arguments to the standard theory of gravity shows that this is not possible in an interesting way: Gravity couples to the spin of fermions via the spin–connection contracted with the spin generators γ_{ab} of the local Lorentz group (since fermions do not transform under general coordinate transformations, but under local Lorentz transformations). Of course, one could make the interaction chiral by contracting the spin–connection with the spin generator of a given chirality (either $\frac{1+\gamma_5}{2}$ or $\frac{1-\gamma_5}{2}$), but then fermions of the opposite chirality would not interact with the spin–connection at all! Therefore, to obtain an interesting theory of chiral gravity one needs two independent spin–connections. This in turn implies that one needs a theory with two dynamically independent vielbeins ${e_{\pm}}_{\mu}{}^{a}$ out of which the spin–connections of the $\pm$ chiralities can be constructed. We conclude that a theory of chiral gravity [5] is necessarily a theory of bi–gravity.

To see how these two seemingly unrelated modifications of conventional gravity come together, we review why it is hard to obtain a covariant completion of the FP mass term. The main reason is that there is only one nontrivial scalar quantity which can be constructed from a metric without using derivatives, namely its determinant. Suppose we start from the simplest possibility,

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 \, R - \Lambda \right] . \tag{2}$$

Expanding the cosmological constant term up to second order in the perturbations $h_{\mu\nu}$ around Minkowski background gives:

$$\sqrt{-\det\left(\eta + h\right)} \;\; = \;\; 1 + \frac{1}{2} \eta^{\mu\nu} \, h_{\mu\nu} + \frac{1}{4} \eta^{\mu\nu} \, \eta^{\alpha\beta} \left[\frac{1}{2} \, h_{\mu\nu} \, h_{\alpha\beta} - h_{\mu\alpha} \, h_{\nu\beta} \right] + \mathcal{O}(h^3) \; . \tag{3}$$

Immediately two problems appear when we compare this expansion with the FP term: (a) The quadratic terms are not those of the FP mass term. (b) The linear term signals that we are expanding around a wrong background (Minkowski rather than de Sitter, as we should be doing for (2)). These observations (and their extension to an arbitrary function of the determinant) led [11] to conclude that completions of (1) cannot be covariant: A background metric ${g^{(0)}}_{\mu\nu}$ has to be combined with the metric $g_{\mu\nu}$ in order to form nontrivial expressions which can reduce to (1) in the weak field limit. [6] We will now show how the chiral theory of gravity introduced above can help resolve these problems.

[5] Chiral gravity should not be confused with spinor gravity [13] in which gravity arises from fermion dynamics.

[6] One way to evade the arguments of [11] is to consider gravity in extra dimensions, and to preserve covariance for the

Let us first address the problem of the appearance of the non–FP quadratic term in (3). Our starting observation is that the FP coefficients are obtained if one considers first order perturbations $F_{\mu\nu}$ in the vielbein ${e_\mu}^a = \delta_\mu^a + F_{\mu\nu}\,\eta^{\nu a}$ (rather than first order perturbations $h_{\mu\nu}$ of the metric):

$$e = \sqrt{-g} = 1 + \eta^{\mu\nu} F_{\mu\nu} + \frac{1}{2}\,\eta^{\mu\nu}\,\eta^{\alpha\beta}\Big[F_{\mu\nu}\,F_{\alpha\beta} - F_{\mu\alpha}\,F_{\nu\beta}\Big] + \mathcal{O}(F^3)\ . \qquad (4)$$

The reason is an extra contribution from the linear term in (3), due to the second order perturbations in the metric

$$g_{\mu\nu} = {e_\mu}^a\,\eta_{ab}\,{e_\nu}^b = \eta_{\mu\nu} + 2\,F_{\mu\nu} + \eta^{\lambda\sigma}\,F_{\mu\lambda}\,F_{\nu\sigma}\ . \qquad (5)$$

Clearly, one could have started directly from the expansion (5), but this would have been a very special and unmotivated choice. We find it remarkable that the FP mass term simply follows from the natural expansion (4) of the determinant of the vielbein.

The remaining issue is how to eliminate the linear term in the expansion (4) when one expands around a Minkowski background. We will show that this can be done by the bi–gravity theory that corresponds to chiral gravity, containing two vielbeins ${e_{\pm\,\mu}}^a$ and metrics $g_{\pm\,\mu\nu}$. This theory consists of a copy of the standard action (2) for each sector with their own Planck mass and cosmological constant. In addition, there is an interaction that couples the two vielbeins in a covariant way via a term which, though similar to, cannot be represented as a determinant. The Minkowski background is not introduced by hand as in [11], but it arises dynamically by a subtle balance between the two gravitational sectors, given by a single fine–tuning of cosmological constant–like parameters. This constitutes the same amount of fine–tuning as for the cosmological constant in the standard theory of gravity.

Combining these ingredients we obtain a theory, that at the linearized level describes two graviton modes which are coupled via a FP mass term. The FP mass can have generic value since the sizes of the cosmological constants are not fixed. The spectrum of the linearized perturbations consists of a massless graviton and a massive graviton, with a FP mass term. (The presence of a single massless graviton is to be expected because the theory of chiral gravity is covariant.) We treat matter in the simplest way possible, that is by coupling it either to the metric $g_{+\,\mu\nu}$ or to $g_{-\,\mu\nu}$; this defines two matter sectors. Since the massive/massless gravitons are linear combinations of the perturbations of the two vielbeins, these mass eigenstates couple to both matter sectors. The interactions of the massless graviton with the two sectors turn out to be equal. The ones of the massive graviton are instead of different strengths and of opposite sign (mediating a gravitational repulsion between the two sectors). The theory possesses interesting limits in which the massless graviton decouples at the linear order, and the massive graviton only couples to one of the two matter sectors. Hence, if we include all the matter in this sector, this limit realizes a covariant nonlinear completion of the FP mass term (1).

The plan of this paper is the following. Section 2 reviews the formulation of standard Einstein gravity in terms of the vielbein and the spin–connection. For this we use a presentation employing Clifford algebra valued differential forms, since it naturally generalizes to the theory of chiral gravity described in Section 3. Readers who are primarily interested in the background evolution and the

standard $3+1$ coordinates. Conventional KK theories have a tower of massive gravitons; however the massless graviton is also present, so that one does not recover a theory of a single massive graviton [14]. Interesting progresses have been recently made in braneworld models. In particular, the model [15] (see also [16] for an interesting earlier attempt) shares many analogies with 4 dimensional models of massive gravity.

linearized spectrum can skip these more technical sections, since the discussion of the final action of chiral gravity in Section 4 is self–contained. The main purpose of this section is to derive and describe corresponding de Sitter and Minkowski background solutions. The spectrum of perturbations around the Minkowski background is computed in Section 5. The coupling of chiral gravity to matter is discussed in Section 6. In particular we pay attention to how the SM can be coupled to this theory. From the coupling of the graviton mass eigenstates to matter we derive effective Planck masses, and limits in which the massless graviton decouples. Section 7 presents our conclusions and some outlook for future work. Appendix A contains our notation and a few technical results.

2 Gravity using the vielbein formalism

We review an elegant representation of the theory of gravity using the vielbein formalism and differential forms. We will use the same formalism to introduce our proposal for a theory of chiral gravity in the next Section. Our conventions and notations have been collected in appendix A. We have followed the presentation of the vielbein formalism of [17], and we have used Clifford algebra valued forms similar to [18]. Our starting point is Einstein gravity with a cosmological constant Λ, described by vielbein and spin–connection one–forms

$$e_1 = e_\mu{}^a\, \gamma_a\, \mathrm{d}x^\mu \ , \qquad \omega_1 = \frac{1}{4}\omega_\mu^{ab}\, \gamma_{ab}\, \mathrm{d}x^\mu \ , \tag{6}$$

which are taken to be independent. The action can be written as

$$S = \frac{i}{4}\int \mathrm{tr}\, \gamma_5 \Big(M_P^2\, e_1^2\, R_2(\omega) - \frac{1}{4!}\Lambda\, e_1^4 \Big) \ , \tag{7}$$

with the Planck mass $M_P^2 = 1/(8\pi G)$, and the curvature associated to the spin–connection

$$R_2(\omega) = \mathrm{d}\omega_1 + \omega_1^2 = \frac{1}{8} R_{\mu\nu}^{ab}(\omega)\, \gamma_{ab}\, \mathrm{d}x^\mu \mathrm{d}x^\nu \ . \tag{8}$$

The equation of motion of the spin–connection is fulfilled, if it satisfies the Maurer–Cartan equation

$$\mathrm{d}e_1 + e_1\omega_1 + \omega_1 e_1 = 0 \ . \tag{9}$$

Writing this equation out in components implies that there is a torsion–free connection, $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$, such that the covariant derivative $\mathcal{D}_\mu$ is covariantly constant on the vielbein:

$$\mathcal{D}_\mu e_\nu{}^a = \partial_\mu e_\nu{}^a - \Gamma^\lambda_{\mu\nu} e_\lambda{}^a + \omega_\mu^{ab}\eta_{bc} e_\nu{}^c = 0 \ . \tag{10}$$

We denote the solution of the spin–connection by $\omega_\mu^{ab} = \omega_\mu^{ab}(e)$, its explicit form is given in (A.4). Substituting this expression back into (10) one finds that $\Gamma^\lambda_{\mu\nu}$ is precisely the conventional Christoffel–connection $\Gamma^\lambda_{\mu\nu}(g)$ defined from the metric (5). The curvature $R^\lambda{}_{\rho\mu\nu}(\Gamma)$ associated to the connection (which is given explicitly in (A.5)) can be expressed in terms of the curvature (8) of the spin–connection as

$$R^\lambda{}_{\rho\mu\nu}(\Gamma) = R_{\mu\nu}^{ab}(\omega)\, (e^{-1})_a{}^\lambda\, \eta_{bc}\, e_\mu{}^c \ , \tag{11}$$

by working out the commutator $[\mathcal{D}_\mu, \mathcal{D}_\nu] e_\rho{}^a = 0$ in components. Therefore, we may refer to the curvature, Ricci–tensor $R^a_\alpha = R^{ab}_{\alpha\beta}(e^{-1})_b{}^\beta$ and the curvature scalar without indicating whether the spin– or Christoffel–connection is used. By employing some Clifford and form algebra, one can show that (7) turns into the standard action for gravity with a cosmological constant (2).

Before we move on to describe our chiral theory of gravity, we would like to make a few comments concerning the symmetries of the standard theory of Einstein gravity in the vielbein formalism: Since the action (7) was written in terms of differential forms only, it is invariant by construction under general coordinate transformations. In addition, the theory is invariant under local $\mathrm{SO}(1,3)$ Lorentz transformations

$$e_1 \to \Omega\, e_1\, \Omega^{-1}, \qquad \omega_1 \to \Omega\,\big(\omega_1 + \mathrm{d}\big)\,\Omega^{-1}\;, \qquad R_2(\omega) \to \Omega\, R_2(\omega)\,\Omega^{-1}\;, \tag{12}$$

with $\Omega = \exp \frac{1}{4}\Omega^{ab}\,\gamma_{ab}$. Finally, we note that the vielbeins are only defined up to a sign, $e_\mu{}^a \to -e_\mu{}^a$ being a symmetry of the theory.

3 Chiral gravity

After the review of the standard theory of gravity in the vielbein and differential form representation, we describe our proposal for a chiral theory of gravity. The Electroweak sector of Standard Model of particle physics is a chiral theory, i.e. the gauge fields couple differently to left– and right–handed states. In the formalism, this is realized by having chiral $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge connections. By analogy, we introduce two vielbeins $e_{\pm\,\mu}{}^a$ and two spin–connections $\omega^{ab}_{\pm\,\mu}$. Since only the spin–connection one–form (6) preserves chirality, as it is proportional to γ_{ab} , we take only the spin–connections to be chiral:

$$e_{1\pm} = e_{\pm\,\mu}{}^a\,\gamma_a\,\mathrm{d}x^\mu\;, \qquad \omega_{1\pm} = \frac{1}{4}\omega_{\pm\,\mu}^{\;\;ab}\,\gamma_{ab}\,\frac{1\pm\gamma_5}{2}\,\mathrm{d}x^\mu\;. \tag{13}$$

We require that these vielbeins and spin–connections transform in the appropriate way under local Lorentz transformations

$$e_{1\pm} \to \Omega\, e_{1\pm}\,\Omega^{-1}\;, \qquad \omega_{1\pm} \to \Omega\,\Big(\omega_{1\pm} + \frac{1\pm\gamma_5}{2}\,\mathrm{d}\Big)\,\Omega^{-1}\;. \tag{14}$$

In principle, we could have required that both $\pm$–sectors have independent Lorentz transformations $\Omega_\pm$. However, this $\mathrm{SO}(1,3)_+ \times \mathrm{SO}(1,3)_-$ gauge symmetry would forbid any interaction between the two sectors, and hence it would not lead to interesting physics. Finally, we generalize the reflection symmetry of the vielbein to both vielbeins $e_{+\,\mu}{}^a$ and $e_{-\,\mu}{}^a$ independently.

In view of the field content, the natural generalization of (7) is given by

$$S = \frac{i}{4}\int \mathrm{tr}\gamma_5\Big\{M_+^2\,e_{1+}^2\,R_{2+}(\omega_+) + M_-^2\,e_{1-}^2\,R_{2-}(\omega_-) - \frac{1}{4!}\Big(\Lambda_+\,e_{1+}^4 - 2\Lambda_0\,e_{1+}^2 e_{1-}^2 + \Lambda_-\,e_{1-}^4\Big)\Big\}\;, \tag{15}$$

where $R_{2\pm}(\omega_\pm) = \mathrm{d}\omega_{1\pm} + \omega_{1\pm}^2$. Here $M_\pm$ can be thought as the analogies of the Planck masses for the $\pm$–sectors of gravity, and $\Lambda_\pm$ and Λ_0 parameterize all possible cosmological constants compatible with local Lorentz invariance and the vielbein reflection symmetries. The sign conventions for the cosmological constants will become clear in the next Section, where we investigate the background solutions. Moreover, we will assume throughout this work that $\Lambda_0 \neq 0$, i.e. that the $\mathrm{SO}(1,3)_+ \times$

$SO(1,3)_-$ Lorentz symmetry is explicitly broken to its diagonal subgroup, which can be identified with the Lorentz group of conventional gravity.

The kinetic terms do not represent the most general form compatible with our symmetries. We restrict ourselves to these structures only, since for them one can immediately give expressions for the spin–connections in terms of the vielbeins, using the solution $\omega_\mu^{ab} = \omega_\mu^{ab}(e)$ encoded in the Maurer–Cartan equation (9) for standard gravity. (The explicit solution is given in (A.4).) In particular, the spin–connection one–forms and the curvature two–forms are given by

$$\omega_{1\pm} = \frac{1}{4}\omega_\mu^{ab}(e_\pm)\gamma_{ab}\frac{1\pm\gamma_5}{2}\mathrm{d}x^\mu \ , \qquad R_{2\pm}(\omega_\pm) = \frac{1}{8}R_{\mu\nu}^{ab}\big(\omega(e_\pm)\big)\gamma_{ab}\frac{1\pm\gamma_5}{2}\mathrm{d}x^\mu\mathrm{d}x^\nu \ . \tag{16}$$

Also in direct generalization of the situation in standard gravity, (5), one can introduce the metrics

$$g_{\pm\,\mu\nu} = e_{\pm\,\mu}{}^a\,\eta_{ab}\,e_{\pm\,\nu}{}^b \ , \tag{17}$$

the Christoffel–connections $\Gamma_{\pm}{}^{\lambda}_{\mu\nu} = \Gamma^\lambda_{\mu\nu}(g_\pm)$ and curvatures $R^\lambda_{\rho\mu\nu}(\Gamma_\pm)$. In particular, we find that the identification between the curvatures in terms of the Christoffel– and spin–connections (11) holds for both sectors separately. This has an important consequence: The first two terms in (15) give rise, aside from the conventional kinetic terms, to a part without γ_5:

$$\pm\frac{i}{4}M_\pm^2\int \mathrm{tr}\, e_{1\pm}^2 R_2(\omega_\pm) = \mp 2i\int \mathrm{d}^4x\,\epsilon^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}(g_\pm) \ , \tag{18}$$

which would make the theory non–unitary. However, this term vanishes because of the cyclicity of the Riemann tensor $R_{\alpha\beta\gamma\delta}(g)$.

To evaluate the cosmological constant–like terms we set up some additional notation. Let $A,\ldots,D$ be four matrices, which like the vielbeins, carry one spacetime and one tangent space index. We define

$$\langle ABCD\rangle = -\frac{1}{4!}\epsilon^{\alpha\beta\gamma\delta}\epsilon_{abcd}A^a_\alpha B^b_\beta C^c_\gamma D^d_\delta \ . \tag{19}$$

The ordering of the matrices $A,\ldots,D$ is irrelevant in this expression, and it generalizes the notion of a determinant, in the sense that $\langle A^4\rangle = \det(A)$. However, $\langle A^2B^2\rangle$ cannot be written as a determinant. The cosmological constant terms in (15) can be cast in the form

$$\frac{i}{4}\frac{1}{4!}\int \mathrm{tr}\,\gamma_5 e_{1+}^p e_{1-}^{4-p} = \int \mathrm{d}^4x\,\langle e_+^p e_-^{4-p}\rangle \ , \tag{20}$$

for $p = 0,\ldots 4$. Similarly to (19), we define

$$\langle ABC\rangle_d^\delta = -\frac{1}{3!}\epsilon^{\alpha\beta\gamma\delta}\epsilon_{abcd}A^a_\alpha B^b_\beta C^c_\gamma \ . \tag{21}$$

In this notation, the Einstein equations of chiral gravity can be compactly written as

$$\frac{1}{4}M_\pm^2\,\epsilon^{\alpha\beta\gamma\delta}\epsilon_{abcd}\,e_{\pm\,\beta}{}^b R_{\pm}{}^{cd}_{\gamma\delta} + \Lambda_\pm\langle e_\pm^3\rangle_a^\alpha - \Lambda_0\langle e_\pm e_\mp^2\rangle_a^\alpha = 0 \ . \tag{22}$$

This representation of the Einstein equations will be our starting point for our study of the perturbations in section 5.

4 Minkowski and de Sitter background solutions

The action of the system can be rewritten in the form

$$S = \int \mathrm{d}^4x \left\{ \sqrt{-\,g_+} \left[\frac{1}{2} M_+^2\, R_+ \;-\Lambda_+ \right] + \sqrt{-\,g_-} \left[\frac{1}{2} M_-^2\, R_- \;-\Lambda_- \right] + 2\,\Lambda_0\, \langle e_+^2\, e_-^2 \rangle \right\} \;, \tag{23}$$

where g_+ (g_-) is the determinant of the metric $g_{+\,\mu\nu}$ ($g_{-\,\mu\nu}$), and R_+ (R_-) its associated Ricci scalar. The relations between the two metrics $g_{\pm\,\mu\nu}$ and the respective vielbeins $e_\pm$ are given in (17). The first two terms describe two separate gravitational sectors, characterized by the Planck masses M_+ and M_-, and the cosmological constants Λ_+ and Λ_-. The two sectors are coupled to each other via the last term (with $\langle \ldots \rangle$ defined in (19)). This coupling cannot be written from the determinant of the two metrics, and this theory is not equivalent to the bi–gravity theories usually considered in the literature [19].

We are interested in background solutions for (23) which generalize the standard Minkowski and de Sitter solutions of Einstein gravity. For this reason, we consider homogeneous and isotropic time dependent vielbeins

$$e_{+\,\mu}{}^a = \mathrm{diag}\big(a_+(t),\, b_+(t),\, b_+(t),\, b_+(t)\big) \;,$$

and analogously for $e_{-\,\mu}{}^a$. From this ansatz, one obtains the following equations of motion

$$\left(\frac{\dot b_+}{a_+}\right)^2 = \frac{1}{3\,M_+^2}\,\left(\Lambda_+\, b_+^2 - \Lambda_0\, b_-^2\right) \;, \tag{24}$$

$$\left(\frac{\dot b_+}{a_+}\right)^{\displaystyle\cdot} = \frac{1}{3\,M_+^2}\,\left(\Lambda_+\, a_+\, b_+ - \Lambda_0\, a_-\, b_-\right) \;, \tag{25}$$

$$\left(\frac{\dot b_-}{a_-}\right)^2 = \frac{1}{3\,M_-^2}\,\left(\Lambda_-\, b_-^2 - \Lambda_0\, b_+^2\right) \;, \tag{26}$$

$$\left(\frac{\dot b_-}{a_-}\right)^{\displaystyle\cdot} = \frac{1}{3\,M_-^2}\,\left(\Lambda_-\, a_-\, b_- - \Lambda_0\, a_+\, b_+\right) \;. \tag{27}$$

where dot denotes differentiation with respect to time. In addition, the non–vanishing components of the spin–connection in the two sectors (defined in eq. (13)) are given by

$$\omega_{\pm\,j}{}^{0\,i} = \frac{\dot b_\pm}{a_\pm}\,\delta_j^i \,, \tag{28}$$

where i, j are spatial indices.

Eqs. (24)-(27) generalize the Friedman equations of standard cosmology. For $\Lambda_0 = 0$ (two copies of the standard case), the two equations (25) and (27) are actually redundant, as a consequence of the Bianchi identities in the two separate sectors (they can be replaced by the equations of state of the fields driving the cosmological evolution, which in the present case are simply $\Lambda_\pm =$ constant). Even in the presence of the mix term, we can still exploit a generalized version of the Bianchi identities, because the two Ricci scalars appearing in (23) are the standard ones. We obtain two simpler equations

than (25)-(27); by differentiating eq. (24) with respect to time, and by combining it with eq. (25), we get

$$\frac{2\,\Lambda_0\,b_-}{a_+}\left(a_+\,\dot{b}_- - a_-\,\dot{b}_+\right) = 0\ . \tag{29}$$

An equivalent equation (wih $+$ and $-$ interchanged) is obtained in the other sector. For $\Lambda_0 \neq 0$ (we also assume that the vielbeins are nonsingular) this enforces

$$\frac{\dot{b}_+}{a_+} = \frac{\dot{b}_-}{a_-} \quad \Rightarrow \quad \omega_+ = \omega_-\ . \tag{30}$$

Hence, the two background spin–connections are equal. This is a very strong constraint, imposed by the rigid structure of (23) and by the simple background considered.

To proceed, we equate the two right hand sides of eqs. (24)-(26), and of eqs. (25)-(27). Combining the two resulting algebraic equations gives

$$a_+\,b_- = a_-\,b_+\ . \tag{31}$$

From the two eqs. (30) and (31) we then find

$$\frac{b_+}{b_-} = \frac{a_+}{a_-} = C\ , \tag{32}$$

where C is a (yet to be determined) constant. As for the spin–connections, cf. eq. (30), the symmetries of the theory force the two sectors to expand with an equal rate. To determine the background solutions explicitly, we specify a gauge for the time variable (analogous to choosing physical or conformal time in standard cosmology). Because $e_{1\pm} = a_\pm(t)\gamma_0 \mathrm{d}t + \ldots$, time parameterizations affect the product, but not the ratio between the two "lapse factors" $a_\pm$. We fix this gauge freedom by setting $a_+\,a_- = 1$, so that

$$a_\pm = C^{\pm\,1/2}\ . \tag{33}$$

In this gauge, the time evolution is encoded by a single function $b\,(t)$, which we define as

$$b_\pm(t) \equiv C^{\pm\,1/2}\,b(t)\ , \qquad H \equiv \frac{\dot{b}}{b}\ . \tag{34}$$

The remaining equations of motion (24) and (26) determine the constant C and the Hubble parameter:

$$\begin{aligned} C &= \left(\frac{M_+^2\,\Lambda_- + M_-^2\,\Lambda_0}{M_-^2\,\Lambda_+ + M_+^2\,\Lambda_0}\right)^{1/2}\ , \\ H &= \frac{\Lambda_+\,\Lambda_- - \Lambda_0^2}{\left(M_+^2\,\Lambda_- + M_-^2\,\Lambda_0\right)^{1/2}\,\left(M_-^2\,\Lambda_+ + M_+^2\,\Lambda_0\right)^{1/2}}\ . \end{aligned} \tag{35}$$

Even though both sectors have de Sitter backgrounds with identical expansion rates, they are not identical, since they are related by a "physical" (in the sense that it cannot be removed by any coordinate reparameterization) conformal rescaling with constant parameter C. We discuss the significance of this rescaling in Section 6, where we introduce matter fields coupled to the two gravity sectors.

A Minkowski background is obtained by a single tuning between the three "cosmological constants" Λ_+, Λ_0, and Λ_- by requiring that the Hubble parameter (35) vanishes. The Minkowski solution is characterized by

$$\Lambda_0 = \sqrt{\Lambda_+ \Lambda_-} \ , \qquad C = \left(\frac{\Lambda_-}{\Lambda_+}\right)^{1/4} \ . \tag{36}$$

Notice that the parameter C in this stationary background does not depend on the values of $M_\pm$. Only for $\Lambda_+ = \Lambda_-$, the backgrounds become identical, $C = 1$. In the next section we investigate the spectrum of perturbations around the Minkowski background characterized by an arbitrary value of C.

5 Perturbation spectrum of chiral gravity

With the results of the Minkowski solution in mind, we investigate the physical spectrum of the theory of chiral gravity in this background. To this end we expand the vielbeins in terms of perturbations $\tilde{F}_\pm$ as

$$e_{\pm\,\mu}{}^a = C^{\pm 1/2}\Big(\delta_\mu^a + \tilde{F}_{\pm\,\mu\nu}\eta^{\nu a}\Big) \ , \qquad \tilde{F}_{\pm\,\mu\nu} = F_{\pm\,\mu\nu} + B_{\pm\,\mu\nu} \ . \tag{37}$$

Metric perturbations are symmetric in the spacetime indices; however, this is not required for the vielbein perturbations. Therefore, we split their perturbations in symmetric $F_{\pm\,\mu\nu} = F_{\pm\,\nu\mu}$ and anti–symmetric $B_{\pm\,\mu\nu} = -B_{\pm\,\nu\mu}$ ones. Using the diagonal Local Lorentz transformations we can require that the anti–symmetric perturbations satisfy $B_{-\,\mu\nu} = -B_{+\,\mu\nu}$. Moreover, one can show that the remaining anti–symmetric tensor perturbations are not dynamical: From the linearized spin–connections

$$\omega_{\pm\,\mu}^{\ ab} = \eta^{a\sigma}\eta^{b\rho}\Big(\partial_\mu B_{\pm\,\sigma\rho} - \partial_\sigma F_{\pm\,\mu\rho} + \partial_\rho F_{\pm\,\mu\sigma}\Big) \tag{38}$$

we infer that the linearized curvatures

$$R_{\pm\,\mu\nu}^{\ ab} = \Big(\eta^{a\sigma}\eta^{b\rho} - \eta^{b\sigma}\eta^{a\rho}\Big)\Big(\partial_\mu\partial_\rho F_{\pm\,\sigma\nu} - \partial_\nu\partial_\rho F_{\pm\,\sigma\mu}\Big) \ , \tag{39}$$

are independent of the anti–symmetric parts $B_{\pm\,\mu\nu}$ of the perturbations. [7] The resulting equations of motion for the remaining anti–symmetric contributions are trivial, and they imply that we can simply put $B_{\pm\,\mu\nu} = 0$. Notice also that both the spin–connections and the curvatures are independent of the conformal factor C.

To read off the spectrum of the theory we substitute these expansions into the Einstein equations (22) and obtain the set of equations

$$\begin{aligned} C\, M_+^2\, G_a^\alpha(F_+) &= \frac{1}{24}\Big(3\Lambda_+ C^2 + 2\Lambda_0 + 3\Lambda_- C^{-2}\Big)\,\mathrm{FP}_a^\alpha(F_+ - F_-) \ , \\ C^{-1}\, M_-^2\, G_a^\alpha(F_-) &= \frac{1}{24}\Big(3\Lambda_+ C^2 + 2\Lambda_0 + 3\Lambda_- C^{-2}\Big)\,\mathrm{FP}_a^\alpha(F_- - F_+) \ . \end{aligned} \tag{40}$$

[7] We have confirmed that the anti–symmetric tensor parts are non–dynamical by computing the quadratic action of the theory as well. In this we differ from the conclusions of [18], where equivalent kinetic terms were considered.

Here the linearized Einstein tensor is given by

$$G_a^\alpha(F) = R_a^\alpha(F) - \frac{1}{2}R(F)\,\delta_a^\alpha \ , \tag{41}$$

where the explicit form of the linearized Ricci tensor is given in (A.6), and the Fierz–Pauli mass operator reads

$$\mathrm{FP}_a^\alpha(F) = 2\Big(\eta^{\alpha\nu}\,F_{a\nu} - \delta_a^\alpha\,\eta^{\mu\nu}\,F_{\mu\nu}\Big) \ . \tag{42}$$

The factor of two in this operator has been included for the following reason: As observed in (11), the curvatures in terms of the vielbein/spin–connection or the metric/Christoffel–connection are equal; therefore, to identify the graviton mass we should use the definition that corresponds to metric perturbations $g_{\pm\,\mu\nu} = C^{\pm 1}(\eta_{\mu\nu} + h_{\pm\,\mu\nu})$. Using the fact that the vielbein perturbations are symmetric, we find the relation

$$h_{+\,\mu\nu} = 2F_{+\,\mu\nu} + F_{+\,\mu\alpha}\eta^{\alpha\beta}F_{+\,\beta\nu} \ , \tag{43}$$

and similarly for the $-$ sector. Hence, to first order, (42) is normalized precisely as a Fierz–Pauli mass term for the perturbations of the metric.

To determine the mass eigenvalues we need to diagonalize the mass terms in (40), while keeping the kinetic terms diagonal. This is achieved by the transformation

$$\begin{pmatrix} F_+ \\ F_- \end{pmatrix} = \frac{1}{r+\frac{1}{r}}\begin{pmatrix} 1 & -\frac{1}{r} \\ 1 & r \end{pmatrix}\begin{pmatrix} F_0 \\ F_m \end{pmatrix} \ , \qquad r = \frac{M_+}{M_-}C \ . \tag{44}$$

The perturbation equations for F_0 and F_m decouple

$$G_a^\alpha(F_0) = 0, \qquad G_a^\alpha(F_m) = m_g^2\,\mathrm{FP}_a^\alpha(F_m) \ . \tag{45}$$

Thus, F_0 describes a massless graviton, while F_m a massive graviton with the FP mass

$$m_g^2 = \frac{1}{3}\,\Lambda_0\,\frac{M_+^2\,C + M_-^2\,C^{-1}}{M_+^2\,M_-^2} \ . \tag{46}$$

We see that the graviton mass m_g vanishes as $\Lambda_0 \to 0$, in which case the the theory describes two massless gravitons (and a massless vector and a massless scalar) as a consequence of the enlarged covariance (as it is manifest in eq. (23)). The mass m_g can be taken to be comparable with the present horizon scale, provided Λ_0 is sufficiently small. It is worth noting that m_g remains small even when one of the two masses M_+ or M_- becomes very large.

6 Matter coupling

We now address the natural question how the matter is coupled to the two gravitons. Our present discussion is far from complete, and it is primarily aimed to determine the effective coupling strengths (Planck masses) of the massless and massive gravitons identified in the previous section by considering

some specific examples. The simplest possibility is to consider two scalar fields ϕ_+ and ϕ_-, which are only coupled to the $+$ and $-$ sector, respectively. The scalar matter action reads concretely

$$S_b = \int \mathrm{d}^4x \left\{ \sqrt{-g_+}\Big[-\frac{1}{2} g_+^{\mu\nu} \partial_\mu\phi_+\partial_\nu\phi_+ - \frac{1}{2} m_+^2\,\phi_+^2 \Big] + \sqrt{-g_-}\Big[-\frac{1}{2} g_-^{\mu\nu} \partial_\mu\phi_-\partial_\nu\phi_- - \frac{1}{2} m_-^2\,\phi_-^2 \Big] \right\} \,, \tag{47}$$

where m_+ and m_- are mass parameters. The scalars are not canonically normalized in the static background (36). This is easily accounted for by the wave function renormalization

$$\tilde{\phi}_\pm = C^{\pm 1/2}\,\phi_\pm \;, \qquad \tilde{m}_\pm = C^{\pm 1/2}\,m_\pm \;. \tag{48}$$

Notice, that the masses $m_\pm$ are also rescaled by the physical conformal factor C. In this way, the energy momentum tensor is identical to the standard one for Minkowski space

$$\tilde{T}_+^{\mu\nu} = \big(-\frac{1}{2}\eta^{\alpha\beta}\partial_\alpha\tilde{\phi}_+\partial_\beta\tilde{\phi}_+ - \frac{1}{2}\tilde{m}_+^2\tilde{\phi}_+^2 \big)\eta^{\mu\nu} + \eta^{\mu\alpha}\eta^{\nu\beta}\partial_\alpha\tilde{\phi}_+\partial_\beta\tilde{\phi}_+ \;, \tag{49}$$

and similarly for the energy–momentum tensor $\tilde{T}_-$ in the $-$ sector.

For the moment we simply assume that our theory contains some form of matter coupled to the $+$ and $-$ sectors encoded in the energy–moment tensors $\tilde{T}_+$ and $\tilde{T}_-$, respectively. If we assume that these forms of matter do not dominate, they are only a source for the two gravitons via the linearized field equations. The precise couplings follow from the diagonalization (44), and they read

$$\begin{aligned} G_a^\alpha(F_0) \quad &= \quad \frac{1}{M_+M_-}\,\tilde{T}_{+a}^{\;\;\alpha} + \frac{1}{M_+M_-}\,\tilde{T}_{-a}^{\;\;\alpha} \;, \\ G_a^\alpha(F_m) - m_g^2\,\mathrm{FP}_a^\alpha(F_m) \quad &= \quad -\frac{1}{CM_+^2}\,\tilde{T}_{+a}^{\;\;\alpha} + \frac{C}{M_-^2}\,\tilde{T}_{-a}^{\;\;\alpha} \;. \end{aligned} \tag{50}$$

Notice that the massless graviton F_0 couples universally to the canonically normalized $+$ and $-$ sectors of the theory. The coupling of the massive graviton F_m is not universal: In fact, because of the opposite sign of the coupling of both sectors, the two sectors feel a repelling force due to the massive graviton.

Hence, generally, the theory of chiral gravity contains both a massless and a massive graviton in its spectrum. Each of them couples to both the $+$ and the $-$ matter sectors, though only the massless one has a universal coupling. It is interesting to consider the limit in which either M_+ or M_- becomes very large. In this limit the massless graviton decouples (at the linear level) from matter, and the massive graviton only couples to the $+$ sector if M_- is taken large, or to the $-$ sector if M_+ is large instead. Hence, assuming that all the matter lives in the sector coupled to the massive graviton, these limits describe a covariant nonlinear completion of the FP mass term. Since only one linear coupling is nonvanishing, we can use it do define an “effective Planck mass” for the (linearized) gravitational interaction. More precisely, the graviton and Planck masses are

$$\begin{aligned} m_g^2 \to \frac{\Lambda_0\,C}{3\,M_-^2} \;, &\qquad M_P \to M_-\,C^{-1/2} \;, \quad \text{as } M_+ \to \infty \;, \\ m_g^2 \to \frac{\Lambda_0}{3\,C\,M_+^2} \;, &\qquad M_P \to M_+\,C^{1/2} \;, \quad \text{as } M_- \to \infty \;. \end{aligned} \tag{51}$$

Notice that in either limit $m_g^2\,M_P^2 \to \Lambda_0/3$.

The introduction of fermionic matter requires more care. A detailed study is beyond the aim of the present work, but some remarks are in order. Fermions with positive chirality are coupled to the spin–connection with positive chirality. For this reason, it is most natural to construct their action only from the + sector of gravity,

$$S_f = \int \mathrm{d}^4x\, e_+\, \bar{\psi}_+ (e_+^{-1})^\mu{}_a \gamma^a \Big(\partial_\mu + \omega_{+\,\mu}\Big)\psi_+ \ , \tag{52}$$

where $\omega_{1+} = \omega_{+\,\mu}\mathrm{d}x^\mu$ is given in equation (13). A similar action can be written down for the negative chirality fermions, which couple to the $-$ sector of gravity. Like the scalars considered above, these fermionic fields are not canonically normalized in the Minkowski background described at the end of section 4. But, by a similar conformal rescaling, this normalization is also obtained for the fermions, so that also their energy–momentum tensors $\tilde{T}_\pm$ are constructed in a similar way to the ones of the scalars.

Severe constraints have certainly to be expected, if the fermions of the SM are embedded in this construction. If SM fermions of different chiralities are coupled to the two different sectors, this would presumably require taking $\Lambda_+ \simeq \Lambda_-$ and $M_+ \simeq M_-$ with high precision. Alternatively, one can construct the Standard Model only starting with one given chirality, and then acting appropriately with the charge conjugation operator. In this way, we can couple all the fermionic matter to one given gravitational sector, and then proceed in analogy to the scalar field case discussed above.

7 Conclusions and outlook

The starting observation of this work is that the Fierz–Pauli tensorial structure is immediately obtained by expanding the determinant e of the vielbein $e_\mu{}^a = \delta_\mu^a + F_{\mu\nu}\eta^{\nu a}$ to second order in the perturbation. This is a quite unexpected result; it had been observed since long that the same does not happen by expanding the metric in its (first order) perturbation, but the simplest case of the vielbein has (to our knowledge) never been noted so far. This observation has led us to address two interesting and seemingly unrelated questions: (i) how one can give a nonlinear completion of the FP term which is covariant, and (ii) how one can construct a theory of gravity that couples differently to positive and negative chiralities.

Let us start from the second one. Since chirality is a property which is naturally connected with local (tangent space) Lorentz transformations, we first reviewed the formulation of standard gravity using the vielbein formalism, based on Clifford valued vielbein and spin–connection one–forms. To obtain a chiral theory of gravity is then straightforward: it requires two independent spin–connections $\omega_\pm{}_\mu^{ab}$ that are contracted with chiral Lorentz generators $\gamma_{ab}\frac{1\pm\gamma_5}{2}$. To ensure that these spin–connections are dynamically independent, two vielbeins $e_{\pm\,\mu}{}^a$ are needed to build to two Einstein–Hilbert terms with Planck masses $M_\pm^2$. In principle there could have been two more kinetic terms that describe the mixing of the spin–connection of one sector with the vielbein of the other. We chose the structure of the kinetic terms such that the standard expression of the spin–connection in terms of the vielbeins could be extended to both sectors separately.

Enforcing reflection symmetry for both vielbeins separately, three different "cosmological constants" could be introduced: in each sector, the conventional one takes the form of a determinant, $-\Lambda_\pm\, e_\pm = -\Lambda_\pm \sqrt{-\,g_\pm}$. The third one mixes the two sectors, $\Lambda_0 \,\langle e_+^2 e_-^2 \rangle$, and cannot be represented as a determinant, see eq. (19). For arbitrary (positive) values of these cosmological constants we found

de Sitter–like solutions in both sectors, with a common expansion rate. However, the scale factors have different off–sets encoded by a parameter C, which is function of the cosmological constants and input Planck masses. A stationary background is obtained by a single fine–tuning: $\Lambda_0^2 = \Lambda_+ \Lambda_-$. In this Minkowski background, the parameter $C = (\Lambda_-/\Lambda_+)^{1/4}$ is not unity in general, hence the conformal off–set remains between the two sectors.

The mix term does not break the covariance of the theory; there is a massless graviton in the spectrum. The remaining degrees of freedom combine in a non–dynamical anti–symmetric tensor, plus a massive graviton. As observed in the introduction, the FP mass term arises naturally when expanding cosmological constant–like interaction in vielbein perturbations. We have finally discussed how matter can be included in this construction, and how it is coupled to the two gravitons. Most relevant for our discussion is the fact that the theory admits interesting limits, in which only the massive graviton is coupled to matter at the linearized level. This construction realizes a covariant nonlinear completion of the FP mass term. It had been previously argued that a covariant formulation is not possible, and that a background (Minkowski) metric has to be used. The introduction of two gravitational sectors provides a way out to this conclusion. An alternative possibility to obtain a covariant theory is to use the Stuckelberg method [9]. As in bi–gravity approaches, the spectrum of the theory is enlarged.

Loosely speaking, the theory of chiral gravity can be viewed as a bi–gravity theory since the set of gravitational fields is doubled. The guideline we have followed, is to construct a model which is as close as possible to the standard theory of gravity. For this reason, we have only allowed for cosmological constant–like terms that can be written down using the two vielbeins (the sets of all possible terms has been further reduced by enforcing a parity symmetry on the vielbeins). This approach can be compared with other constructions were bi–gravity theories have been used to complete the FP mass term; these models are typically characterized by a large arbitrariness of the choice of potentials for the two gravitons, see for instance [19–21]. Although it is fair to say that also our criteria do not select a unique theory (and so, do not allow for a predictive effective field theory, in the sense mentioned in the Introduction), the model (23) is an immediate and simple generalization of the Einstein–Hilbert term of standard gravity. Whether this may be of any practical advantage over other choices requires to discuss the theory beyond the nonlinear level (we hope to come back to this point in a separate publication).

The action (23) appears to be particularly simple due to the use of vielbeins rather than the two metrics. One could in principle try to express the interaction $\langle e_+^2\, e_-^2 \rangle$ in terms of the two metrics. This would however lead to a rather involved expression without any clear motivation. On the other hand, the arguments we presented together with eq. (3) show that a simple generalization of the cosmological constant term is unlikely to have the FP limit at the linear level. This is a strong motivation for the use of the vielbeins rather than the metric in the present construction.

The equivalence between chiral and the more often studied theories of bi-gravity (formulated using two metrics) is not trivial. The two vielbeins have also anti–symmetric components which, as long as the two sectors are decoupled, can be removed by two independent local Lorentz transformations. The mixing term $\langle e_+^2\, e_-^2 \rangle$ is invariant only under a combined Lorentz transformation, therefore one of these anti–symmetric tensors cannot be removed any longer. The perturbative calculations of Section 5 show that this field is not present in the quadratic action for the perturbations. We expect that this is also the case at the non–linear level. Indeed, a variant of the Stückelberg formalism (along the lines of [9]) could be employed to confirm that this anti–symmetric tensor is not dynamical at any order.

The potential presence of this additional field was already noted in the work [18], which bears

similarities with our construction. Ref. [18] considered a complex vielbein, which we found to be equivalent to the chiral ones we have introduced. In [18], it was claimed that the anti–symmetric combination gives a dynamical field already at the quadratic level. Unfortunately, the formulation given in [18] is much less tractable then the one we presented, since the kinetic terms for the two different degrees of freedom do not appear to be decoupled; for this reasons, the solutions in [18] have been given only perturbatively, and we believe that the claim that the anti-symmetric field is physical (at the linear level) is erroneous. [8]

There are several open issues left for future investigation. For instance, we did not compute any experimental limit if different types of matter are coupled to the two different gravitational sectors (see however the discussion at the end of Section 6). We also neglected the influence of matter on the evolution of the background. Computing cosmological solutions in presence of matter could help addressing the main motivation for massive gravity, namely the present acceleration of the universe. In this respect, it is worth noting that the massive graviton in our model can also mediate a gravitational repulsion between different types of matter. However, from what we already argued, it is clear that the most relevant open questions are related to the nonlinear behavior of gravity in this theory. We discussed in details the spectrum of linearized perturbations around Minkowski background, showing how the massless graviton decouples in certain limits. At the linear level, we have thus only massive gravity of the FP form. However, the nonlinear structure is now richer, and it could shed some light on open issues of massive gravity. Computations of nontrivial backgrounds, for instance the generalization of the standard black–hole solutions, may also provide important information in this regard.

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A Technical details

We use the mostly plus convention for the metric, in particular the Minkowski metric η_{ab} reads $\mathrm{diag}(-1,1,1,1)$. We use indices $a, b, \ldots$ for the tangent space, and $\alpha, \beta, \ldots$ to denote spacetime indices. In particular, we use differentials $\mathrm{d}x^\mu, \mathrm{d}x^\nu, \ldots$ that anti–commute

$$\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}x^\nu \mathrm{d}x^\mu \ , \qquad \mathrm{d}x^\alpha \mathrm{d}x^\beta \mathrm{d}x^\gamma \mathrm{d}x^\delta = \epsilon^{\alpha\beta\gamma\delta} \, \mathrm{d}^4 x \ , \tag{A.1}$$

where $\epsilon^{\alpha\beta\gamma\delta}$ is totally anti–symmetric, with $\epsilon^{0123} = 1$. In the definition of the forms we include appropriate symmetrization factors. For example, for a two–form we write $B_2 = \frac{1}{2} B_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu$.

The Clifford algebra is generated by γ_a that satisfies

$$\{\gamma_a, \gamma_b\} = 2\,\eta_{ab} \ , \qquad \gamma_{ab} = \frac{1}{2}[\gamma_a, \gamma_b] \ , \qquad \gamma_5 = i\,\gamma_0\gamma_1\gamma_2\gamma_3 \ . \tag{A.2}$$

It follows that

$$\mathrm{tr}\,\gamma_5\gamma_a\gamma_b\gamma_c\gamma_d = 4i\,\epsilon_{abcd} \ , \tag{A.3}$$

[8] In addition, the general relation between the FP term and the expansion (4) is not manifest (nor noted) in [18].

where ϵ_{abcd} is totally anti–symmetric with $\epsilon_{0123} = -1$.

The spin–connection is given in terms of the vielbein as

$$\begin{aligned} \omega_\rho^{ab}(e) &= -\tfrac{1}{2}\Big\{\partial_\mu e_\nu{}^c\,\eta_{cd}\,e_\rho{}^d + \partial_{[\mu}e^c_{\rho]}\,\eta_{cd}\,e_\nu{}^d - (\mu\leftrightarrow\nu)\Big\}\,\eta^{am}\,\eta^{bn}\,(e^{-1})_m{}^\mu(e^{-1})_n{}^\nu\ , \\ R^{ab}_{\mu\nu}(\omega) &= \partial_{[\mu}\omega^{ab}_{\nu]} + \omega^{ac}_{[\mu}\eta_{cd}\omega^{db}_{\nu]}\ . \end{aligned} \tag{A.4}$$

In the second line we have given the component form of the curvature defined in (8). The Christoffel–connection and curvature read

$$\begin{aligned} \Gamma^\lambda_{\mu\nu}(g) &= \tfrac{1}{2}g^{\lambda\rho}\Big\{-\partial_\rho g_{\mu\nu} + \partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho}\Big\}\ , \\ R^\lambda{}_{\rho\mu\nu}(\Gamma) &= \partial_\mu\Gamma^\lambda_{\nu\rho} - \partial_\nu\Gamma^\lambda_{\mu\rho} - \Gamma^\kappa_{\mu\rho}\Gamma^\lambda_{\nu\kappa} + \Gamma^\kappa_{\nu\rho}\Gamma^\lambda_{\mu\kappa}\ . \end{aligned} \tag{A.5}$$

Finally, the linearized Ricci tensor in the vielbein formalism is expressed as

$$R^\alpha_a(F) = \partial_a\partial^\rho(F_{\rho\sigma}\eta^{\sigma\alpha}) + \partial^\rho\partial^\alpha(F_{\rho a}) - \partial_a\partial^\alpha(F_{\rho\sigma}\eta^{\rho\sigma}) - \partial_\rho\partial^\rho(F_{a\sigma}\eta^{\sigma\alpha})\ . \tag{A.6}$$

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